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The breakup of a turbulent liquid jet in a gaseous atmosphere

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An electrical method of detecting and measuring the breakup of liquid jets is applied to the turbulent case. New data produced by this means, together with previous data, support the conjecture that the theory and understanding that were developed in connexion with the breakup of laminar jets can be used as a guide for turbulent jets as well.

1. Introduction

The breakup of turbulent liquid jets is frequently of technical importance. Familiar examples of applications are streams from fire hoses, irrigation nozzles or fuel nozzles. In spite of their importance, there is little understanding of the breakup mechanism, no general theory, nor even a reasonably successful method of correlation. Very little experimental data finds its way into the scientific literature owing to this lack of any theoretical framework into which to fit it.

The work described below supports the notion that laminar and turbulent liquid jets have many similarities. We attempt to apply the vast literature and the greater understanding of laminar jets to the problem of the breakup of turbulent jets. In particular, the possibility that the breakup mechanisms are basically the same for laminar and turbulent jets is explored; the primary differences are in detail. The most obvious difference is in the disturbance level at the jet exit, which is of course much higher for turbulent jets.

To put the discussion of turbulent jets into proper perspective, we first review the corresponding situation for laminar jets.

2. Background

If the experimentally determined breakup length of a jet is plotted against the average exit velocity, a curve similar to that shown in figure 1 is obtained. The laminar linear portion is well understood. It is described very well by Weber's (1931) theory, which includes the diameter d, surface tension σ, jet viscosity μ and density ρ_j; but which ignores the ambient density ρ_a. At some point, the linear theory fails, and there is a peak in the breakup curve. The reasons for the peak are only now becoming clear. The peak can, in fact, be caused by two different effects. Initially, they can be considered as separate, but later on it will be seen how these effects combine.

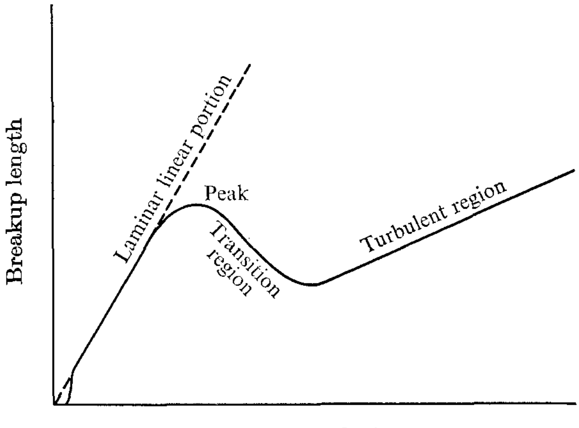

FIGURE 1. Typical breakup length *vs.* velocity curve.

The first factor is the variation of the initial disturbance level with the exit velocity. It is dealt with in detail in a previous publication by the author (1972) and is summarized here. Even if the ambient density is low enough to have no influence on the jet breakup, we still find that there is a peak in the curve of figure 1. The explanation does not seem to be that there is a new mode of instability that is shortening the breakup length, but that the exit disturbance level itself is increasing while the amplification rate is unchanged from that given by the surface tension mechanism assumed in Weber's theory.

The stability parameter $\lambda = (L/d)/We_j^{\frac{1}{2}}(1+3Z)$ is used to characterize breakup. Depending upon the point of view that is taken, it can be considered as the reciprocal of the amplification rate for disturbances or as the logarithm of the initial disturbance level. In the parameter λ, L is the breakup length, d is the jet diameter and We_j is the jet Weber number $d\rho_j V^2/\sigma$, where V is the exit velocity and σ is the surface tension. The amplification rate is that given by Weber's theory, in which the destabilizing force is the contraction of surface tension, which tends to pinch the jet. The factor $1+3Z$ contains the effect of viscosity through the Ohnesorge number $Z = \mu/(\rho_j \sigma d)^{\frac{1}{2}}$, where μ is the jet viscosity. This factor accounts for the stabilizing influence of viscosity, but will not differ significantly from one for the turbulent jets considered here.

It is found that the Reynolds number $Re = Vd\rho_j/\mu$ of the flow through the nozzle is the parameter that controls the disturbance level in a jet. If curves of λ *vs.* Re are plotted for various nozzle–fluid combinations (the influence of the ambient atmosphere being negligible), it is found that for each curve there is a critical Reynolds number $\hat{Re}$ above which λ starts to decrease rapidly. This is interpreted as an increase in the exit disturbance level. This critical value $\hat{Re}$ at which the increase takes place is of the same order as, but below, that for transition to turbulence in the nozzle. On physical grounds this is plausible. The details of the transition region are unknown, but one is led to speculate that $\hat{Re}$ is a mild but consistent function of the fluid properties through the parameter Z.

The second factor that produces a peak in the curve in figure 1 is the influence of the ambient atmosphere. In this case, we consider the initial disturbance level as constant ($Re < \hat{R}e$) and examine the effect of the ambient density. A theory that includes the ambient density through the Weber number $We_a = dV^2\rho_a/\sigma$ is reasonably able to predict experimental results (see Phinney (1973) for details). Physically, the breakup mechanism is due to the relative motion of the jet and the ambient fluid, which induces a surface pressure that tends to increase any non-uniformities of the jet surface. The theory is just an improvement of that of Weber that includes the factor We_a. The limit $We_a \rightarrow 0$ corresponds exactly to the analytic result obtained by Weber. When the theoretical curves are normalized with respect to their $We_a = 0$ asymptotes, a nearly universal curve is obtained for all moderately low values of Z (nozzle–fluid combinations). As We_a increases, there is a 'break' in these normalized curves for We_a of order one. When experimental values of λ are plotted against We_a, a good correlation is found. Again, the parameter λ is found to be the important non-dimensional parameter.

Since in both of the above cases it was found that the parameter λ is of prime importance in the interpretation of the experiments, we shall carry this parameter over to the turbulent case. It is seen that λ can be thought of as including both the initial disturbance level and the ambient atmosphere. It also acknowledges the fact that these two factors are usually inseparable in experiments.

3. Outline of the method for turbulent jets

Turbulent flows have velocity fluctuations that are large enough that they usually cannot be considered as small perturbations from a steady mean flow. This is the main objection to using a linearized laminar stability theory for turbulent jets. If the theoretical analysis does not apply directly, one is tempted at least to see if the same mechanisms and the same non-dimensional parameters can be carried over to the turbulent case.

If plots of λ *vs.* Re (Phinney 1972) are extended to turbulent Reynolds numbers, we obtain the results shown schematically in figure 2. The left-hand portion of the figure shows the well-known laminar plateau on which the amplification rate and the disturbance level are constant (the limit of both We_a and Re small). Also, shown by dotted lines are the curves that are obtained with high ambient density, so that We_a is of order one or greater.

As was noted previously, there is a transition region somewhere in the range $500 < Re < 3000$ where the disturbance level increases sharply. Above the transition, there is a fully turbulent regime. In this regime, one gets the impression that a second plateau may exist with a constant but higher disturbance level. On the basis of measurements of the velocity fluctuation levels in turbulent pipe flow, one would expect a relatively constant disturbance level over a broad range of exit velocities. The existence of a second plateau would not be surprising from this point of view. What might be surprising is that the amplification rate as calculated from laminar theory could be applied to this case with any success.

By analogy with laminar jets, the possibility that the ambient atmosphere will have an influence must be expected. If the Weber number We_a based on the

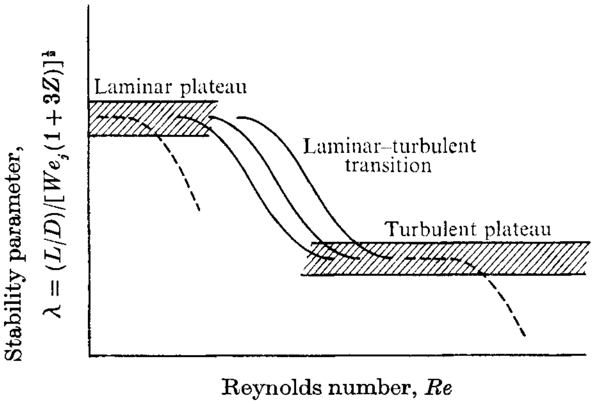

FIGURE 2. Schematic diagram of the variation of the breakup-length parameter λ with Reynolds number. – – –, effect of ambient atmosphere.

ambient density is of order one, the amplification rate for disturbances should increase and the jet length shorten. This is shown schematically in figure 2 by a dotted line which falls away from the constant disturbance level plateau. The high exit velocity of turbulent jets makes it very rare that We_a is significantly less than one, so that the influence of the ambient atmosphere must be considered for all the data.

In the study of laminar jets, there are sufficient data so that the two effects, (i) the increase in exit disturbance level and (ii) the increase in amplification rate due to the ambient atmosphere, can be studied separately. Cases for which both effects occur simultaneously have not been examined to see if theory properly accounts for these data. However, since the theory seems satisfactorily to explain the effects separately, it should also account for them simultaneously. For the turbulent case, where the two effects occur together, we are forced to assume that their influences on the breakup length combine (as they should for laminar jets) into a product of a disturbance level and an amplification rate term. Initially, there seems little concrete justification for this assumption. The real justification is, as we shall see, that it appears to work. To verify the proposed method rigorously would require much more test data than are available. In particular, it would be necessary to perform systematic experiments with reduced ambient pressures in order to separate better the effects of the disturbance level and ambient atmosphere.

Let us pause before considering the details of the analysis in which we try to effect the separation discussed above. The next section looks at the sources of data that will be used in connexion with the analysis.

4. Discussion of available data

Recently some experiments with turbulent liquid jets have been performed at the Naval Ordnance Laboratory (NOL). These experiments extend the range of some earlier work previously reported by Phinney & Humphries (1970). An

User	Designation	ρ (g/cm^3)	μ (centipoise)	σ (dyne/cm)	Ambient density (atm)
Chen & Davis	None	1·0	1 (nominal)	72 (nominal)	1·0
UTC	1	1·0	1	72	1·0
	2	1·0	1	72	2·05 (freon 114)
	3	1·0	1	72	4·26 (freon 114)
	4	1·0	1	72	8·13 (freon 114)
NOL	I	1·106	1·34	77·5	1
	II	1·116	1·41	37·4	1

TABLE 1. Fluid properties

electrical technique for measuring and recording jet breakup was used. It is similar to the method described by Vereshchagin, Semerchan & Sekogan (1959). The water-based fluid is made conducting by the addition of table salt in order that continuity can be detected by electrical means. The measurements are made to be independent of the jet resistance by using a gate circuit that passes pulses when the current through the jet is broken and not passing them when it is continuous. By moving the electrical pickup point along the jet, the percentage of the time that the jet is continuous can be measured at various points. A position can be found where the jet is continuous 50 % of the time. This position is defined as the average breakup length L for this experimental configuration. The details of the experimental apparatus are given by Phinney & Humphries (1970).

The advantage of the electrical method, over the usual photographic one, is that many measurements can be made quickly and accurately and can be averaged automatically. This is especially advantageous for turbulent jets, where it is hard to define in a photograph where the breakup point is located. In the present study, these new NOL data were used to establish a basic correlation. Then data from two other sources were used to verify the results. This approach was taken since the exact fluid properties of the NOL tests were known to the author and because there are a large number of points (actually about half the NOL data are left off figures 3 and 5 owing to crowding) that define the curves very well. Generally the NOL data also had low enough values of We_a to establish a low-speed asymptote, whereas the data from the other sources had relatively large We_a.

To the authors' knowledge, there is very little sufficiently well-defined data in the literature concerning the breakup of turbulent jets that can be used in the evaluation of a theory. Chen & Davis (1964) report some measurements that were made with large diameter plastic pipes. Some other data were made available privately by Dr R. Dunlap of the United Technology Center (UTC).

The characteristics of the fluids and of the nozzles used in the present study are given in tables 1 and 2, respectively. In cases where the measured values (if they were measured) of fluid properties from the other laboratories are not known, the handbook values are entered in table 1.

User	Designation	Diameter (cm)	Length/ diameter	Material
Chen & Davis	$\frac{1}{4}$	0·635	100	Plastic
	$\frac{3}{8}$	0·9525	100	Plastic
	$\frac{1}{2}$	1·27	100	Plastic
	$\frac{3}{4}$	1·905	100	Plastic
UTC	None	0·0343	6·89	Steel
NOL	*A*	0·493	103·5	Glass
	B	0·206	103·1	Glass
	C	0·125	102·7	Glass
	D	0·0504	103·5	Glass

TABLE 2. Nozzle characteristics

Some other comments concerning the tables are in order. All the experiments were performed with a jet fluid that was basically water. Most of the variation in parameters was due to nozzle diameter changes. In the experiments at UTC the ambient density was varied through a range of 1–8 by using freon 114 at various pressures. Although there is not much data available from this source and the nozzle has a relatively low length-to-diameter ratio, it is nevertheless included because it is only set that includes variations in the ambient density.

In the NOL test, the exit velocity was determined by time volume flow measurements as a function of the nozzle pressure. During the breakup-length measurements, the nozzle pressure was noted for each point and the corresponding exit velocity was then found by interpolation on the pressure–velocity plot. The fluid viscosity was measured by flow through a capillary tube, the density by weighing a fixed volume, and the surface tension with a du Noüy Tensiometer. The accuracies of the fluid properties and nozzle diameters are easily within 1 %.

5. Analysis of the data

When all the available data are presented in the same form as in figure 2, one sees a second plateau in rough form. Figure 3 shows the data corresponding to tables 1 and 2 plotted in this way. Closer examination shows that there are some systematic deviations from the simple constant level plateau. The NOL data because of the large number of closely spaced points bring out these features most clearly. In particular, note the following departures from the 'ideal' behaviour.

(*a*) After the transition from the laminar portion of the curve, there is a turbulent plateau followed again by a falling part of the curve.

(*b*) The turbulent plateau is not quite the same for any of the nozzle–fluid combinations.

Because the plateau merges to some extent with the transitional region and because low enough velocities ($We_a \ll 1$) are not available for all curves, there are some problems connected with obtaining a clean separation of the effects of the disturbance level and ambient atmosphere. The approach that is taken is the following. For each curve an 'asymptotic' value λ_1 was defined. It is the

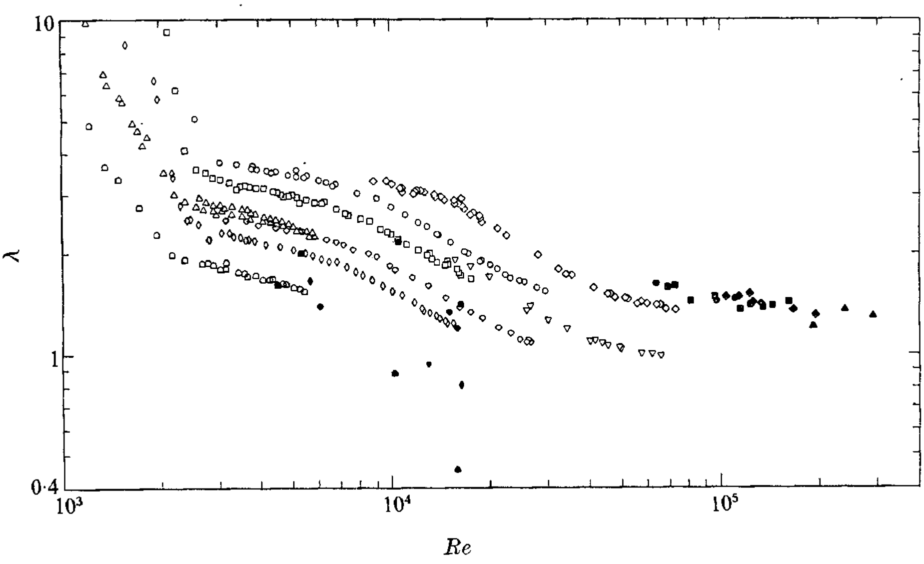

FIGURE 3. Breakup-length data as a function of Reynolds number. NOL: ◇, *A*I; ○, *B*I; □, *C*I; △, *D*I; ▽, *A*II; ▽, *B*II; ◊, *C*II; ⌓, *D*II. Chen & Davis: ●, $\frac{1}{4}$ in.; ■, $\frac{3}{8}$ in.; ◆, $\frac{1}{2}$ in.; ▲, $\frac{3}{4}$ in. UTC: ●, 1 atm; ●, 2·0 atm; ◆, 4·26 atm; ▲, 8·13 atm.

value of λ corresponding to an exit velocity with $We_a = 1$. In practice this was done by fairing through the data and possibly extrapolating it to a small extent to lower We_a. λ_1 is not truly an asymptotic value, but at least it characterizes the low We_a portion of the curve, where it is presumed that the ambient atmosphere has negligible influence. By analogy with the laminar theory, the 'asymptote' λ_1 is interpreted as defining the exit disturbance level. When plotted as in figure 4, the 'asymptote' or 'disturbance level' is found to have a systematic variation with the Ohensorge number Z. The justification for the choice of Z as the controlling parameter is that it is the only non-dimensional characteristic (excluding the ambient density since it is assumed to have no influence on the exit disturbance level) of the nozzle–fluid combination. The present data seem to correlate with this choice, at least for the limited data available. The slight variation of the asymptote with Z can also be seen for laminar data as summarized by Phinney (1972). These corresponding asymptotes for the laminar jets are also included in figure 4 for comparison. It is seen that there is a certain correspondence between the turbulent and laminar data. The turbulent data are much lower, of course, owing to the larger exit disturbance level. The fact that the laminar curves are displaced with respect to one another is undoubtedly due to the variation in exit disturbance level from one set of laboratory apparatus to the next.

By dividing the values of λ for each nozzle–fluid combination by the corresponding 'asymptotic' value λ_1, normalized curves are obtained. These normalized curves should show the influence of the ambient atmosphere when plotted against $We_a^{\frac{1}{2}}$. This is the same technique as was employed by Phinney (1973) in the study of laminar jets. Figure 5 shows the results of plotting the normalized curves. The data for $We_a^{\frac{1}{2}} > 0{\cdot}7$ seem to correlate quite well. For $We_a^{\frac{1}{2}} < 0{\cdot}7$,

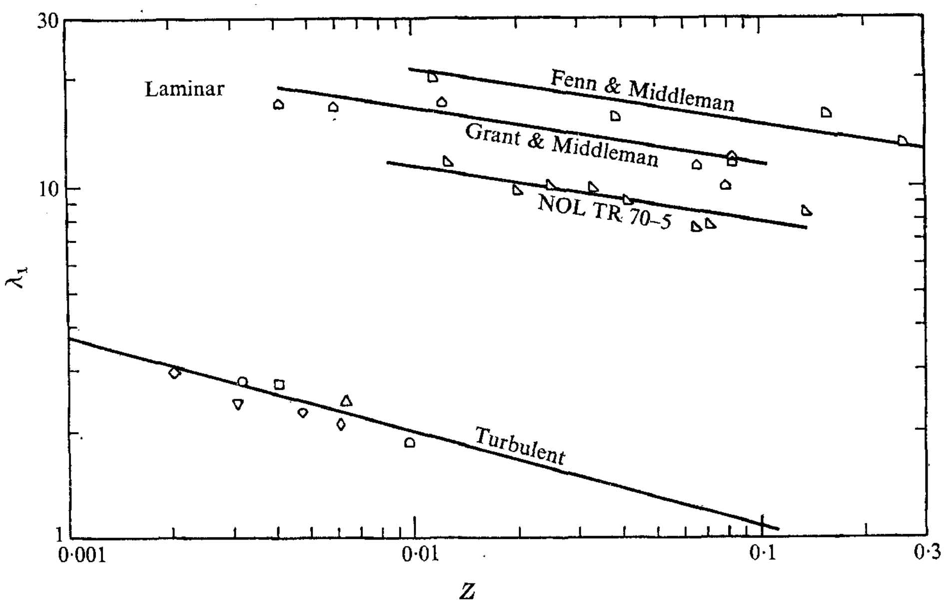

FIGURE 4. Asymptotic values of λ plotted against Z. Symbols as in figure 3.

the large scatter and the large values of λ correspond to the fact that these points are already in the transitional regime and cannot be expected to correlate on the basis of a fully turbulent jet.

As a further test of the proposed method of analysis, the data from two other sources are also presented in the same format. Owing to the lack of a sufficient number of data points for the data of both UTC, and Chen & Davis (1964), the low We_a plateau was more difficult to establish. In order to overcome this problem, the NOL correlation line in figure 4 was used to obtain λ_1 from the known value of Z for each of these other nozzle–fluid combinations. With these indirectly determined values of λ_1, the curves of UTC and Chen & Davis are normalized and included in figure 5. These data fit quite well considering the fact that the normalizing factor λ_1 was determined from the NOL data and not adjusted for each curve to give the best fit with the other data in figure 5.

For comparison purposes, two curves are drawn in figure 5. The solid line is the theoretical curve for laminar jets with small Z, and is taken from Phinney (1973). The dotted curve represents the mean of Fenn's data, taken from the same source. It is concluded from figure 5 that the influence of the ambient atmosphere can be described by the parameter We_a. Although the functional relationship between λ and We_a is not the same as it is for laminar jets, a critical We_a of order one still seems to apply. The limited data of UTC help to establish the importance of We_a since in this case the ambient density is varied directly, and the results are observed to correlate on the basis of We_a.

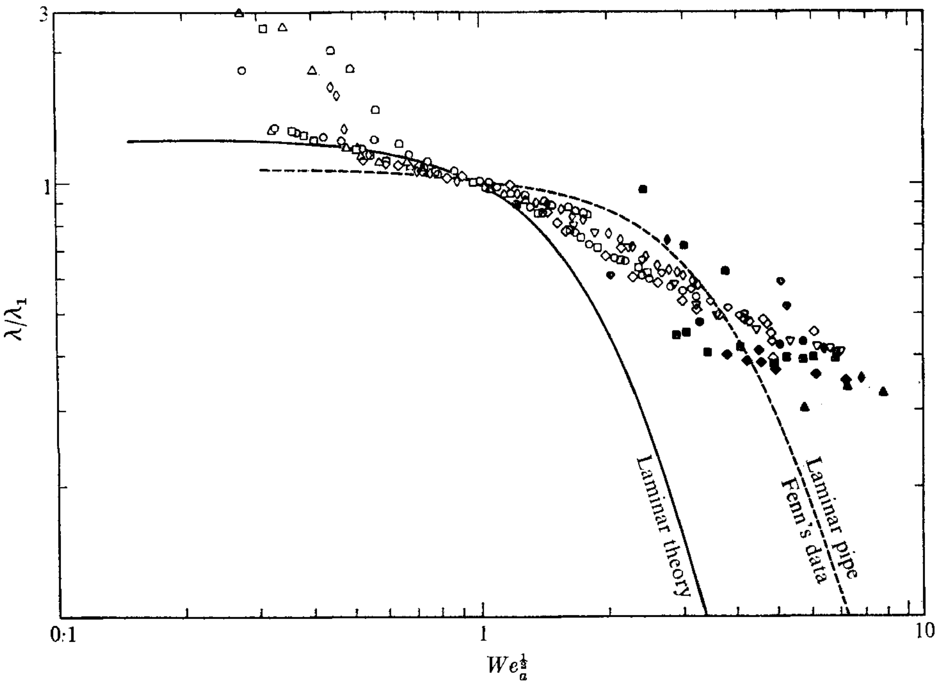

FIGURE 5. The influence of the ambient atmosphere. Symbols as in figure 3.

6. Some results and conclusions

An advantage of the electrical method of measurement is that it becomes possible to obtain easily statistical information concerning the variation in the breakup point along the jet. For a fixed jet velocity, the pickup screen is moved along the jet, and the mean percentage of time that the jet is continuous is recorded together with the distance of the screen from the jet exit L (%). The mean breakup length L corresponds to L (50 %). When these statistical data are presented in Gaussian probability co-ordinates with the parameter L (%)/L plotted against the percentage of the time that the jet is continuous, figure 6 results. In the co-ordinates shown, the data for four nozzle–fluid combinations at several Reynolds numbers all seem to collapse into a straight line. The Reynolds number is given in the legend to identify the jet exit velocity. It is not clear that any systematic trends away from the linear relationship exist in figure 6, although there may be a tendency for the higher exit velocities to have slightly lower slopes. If there is a systemetic variation in slope, it is not clear what parameters should control it. Some preliminary measurements of this kind were reported by Phinney & Humphries (1970). These measurements were for laminar and transitional jets, as well as for non-Newtonian fluids. All of those data agree quite well, as to slope, with the data in figure 6 for turbulent Newtonian jets.

Needless to say, the statistical variation of the breakup length seems to be described quite well by a Gaussian distribution, as shown by figure 6. The Gaussian

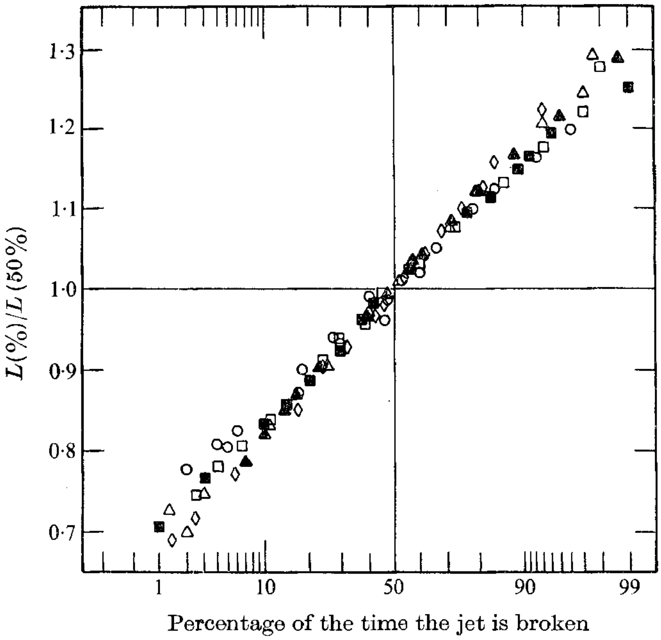

FIGURE 6. Statistical variation of breakup point along the jet.

	○	□	■	△	▲	◇
NOL fluid	*AI*	*BI*	*BI*	*CI*	*CI*	*DI*
	22000	8500	21000	5400	7800	3200

distribution is symmetric about the 50 % point L. It follows that L is the most probable position at which new breaks in the jet will form. This gives a basis upon which to expect that the electrical definition of L as the breakup length should coincide with the photographic definition, in which the mean position for fresh breaks is used to define the breakup length.

In addition to the electrical measurements of the breakup length, some optical observations were also made. These observations were made visually and not photographically, so that they are somewhat subjective. On the other hand, the surface of the jet is three-dimensional, so that it is sometimes easier to see things at first hand rather than in photographs. Also, the selection and interpretation of photographs are somewhat subjective in themsevles. With the above caution, some tentative conclusions can be drawn.

For each nozzle tested, visual observations were made for the same velocity range as was used in the breakup-length tests. Strobe lighting that gives a sequence of instantaneous 'pictures' was used as well as room light. Under ordinary illumination, the breakup mechanism of the jet cannot be seen, but it is possible to see the increase in exit disturbance level. For laminar flow, the jet appears as a smooth silver thread. Sometimes, if the light is reflected properly and the strobe frequency is right, regular instability waves can be seen owing to the slight variations in jet diameter. The turbulent jet appears milky white at the exit, and no further details are seen. Under strobe lighting, the reason for the milky appearance is obvious. The surface is again seen as shiny, but it is completely covered with small random protuberances (that diffusely reflect the light) caused by the turbulent motion inside the jet. Surface tension tends to reduce these sharp bumps as the jet proceeds away from the exit; however, the rough surface is visible up to the point of breakup. No basic change is seen in the breakup pattern for the range of turbulent jets studied here. The final breakup itself was always in the form of droplets whose diameter was of the same order as the jet diameter (if the drops have sufficient velocity, they can suffer further breakup, of course). Although the breakup process did not produce perfectly aligned droplets, it is questionable whether the motion can be characterized primarily as sinuous or serpentive, in which ligaments are stretched laterally with increasing amplitude. The present observations suggest that an alternative explanation is possible. What appears as a sinuous motion to some could equally well be viewed as an unsymmetrical turbulent motion that produces lateral motion of the jet, but is not amplified and does not contribute to the jet instability process. This is illustrated by the turbulent mercury jet of Merrington & Richardson (1947). The pictures that they present in their figures 10–12 are typical of the NOL observations of turbulent water jets. In their photographs, it is difficult to see any regular sinuous wave structure. When viewed in three dimensions, there is even less evidence of a regular wave. According to Merrington & Richardson, the sinuous motion is observed near the exit, but has a decreasing amplitude until 'the varicose motion eventually persists and breaks up the jet'.

Although some authors believe that a sinuous mode of instability is necessary to explain the breakup of high-speed jets, this can be countered by the following arguments. First, the disturbances that are supposed to be sinuous waves do not appear to be orderly. They are not in a plane or 'corkscrew', in fact, the peaks are oriented in random directions, suggesting a turbulent origin rather than the growth of a regular instability. Another reason to be suspicious of the possibility of a new mode of instability is that, although there have been attempts, no plausible physical mechanism for the amplification of these disturbances has been advanced which shows a correlation with experimental data. Yet another reason to be suspicious is the success of the present correlation of turbulent jets. The method is based upon the parameter λ, which implicitly assumes an axisymmetric disturbance amplified by the surface tension mechanism of Weber (1931) (which includes the influence of the ambient atmosphere).

7. Concluding remarks

The fact that there has been no theroretical background into which to fit experimental observations has blocked progress in understanding the breakup of turbulent jets and discouraged general experimental studies. The present work suffers to some extent from this lack of a general fund of experimental data. The purpose of this study is to report some recent experiments and show how they can be correlated with other data on a quasi-theoretical basis. A secondary objective is to demonstrate the advantages of the electrical system for detecting the breakup of turbulent jets. The present approach leaves some questions unanswered, but hopefully gives a starting point for future studies.

The 'theory' used here is an extension of laminar jet stability theory. By analogy with the laminar experiments, the influence of the initial disturbance level and the ambient atmosphere can be identified. For turbulent jets, the constant disturbance level plateau (for low We_a) is inferred but has not been completely isolated since the necessary experiments with low ambient pressure have not been performed. On the basis of the data in figure 5, the asymptote for low We_a does seem to exist.

It is also concluded that the low We_a plateau level is not an absolute constant, but depends to some extent on Z as in the laminar case. This dependence on Z must also be considered as a tentative conclusion on the basis of limited data. It appears to be reasonably successful, on the basis of the fact that data from all sources seem to follow the trends established by the NOL data. The initial disturbance level for turbulent jets probably displays less variation from laboratory to laboratory than it does for laminar jets. If so, it would not be surprising since the disturbance level for turbulent jets is largely independent of the apparatus and is fixed by the turbulence mechanism itself; whereas, on the contrary, laminar jets depend upon such things as pipe smoothness, building vibration and a host of other variable causes.

For long nozzles, the turbulence level at the exit reaches an equilibrium value that does not depend upon length. When the nozzle is short the equilibrium value is not reached, and the jet breakup characteristics can deviate from the long nozzle results. As nearly as possible, this question was avoided by testing nozzles of sufficient length (length/diameter > 100) such that equilibrium was reached. The exception is the UTC data, where the length-to-diameter ratio was of the order of seven. In spite of this shortcoming, the data were included since they are the only data available that include variations in ambient density.

As a word of caution, it should be noted that the present experiments have relatively low turbulent Reynolds numbers and that We_a does not exceed one by more than an order of magnitude. At much higher exit speeds or ambient densities, the breakup mechanism may undergo a transition to some other mode that was not experienced in these tests. For example, there was negligible striping of the surface layer from the jet due to the relative motion with the ambient atmosphere. At high exit speeds, this effect might dominate the breakup process studied here.

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